Precalculus Honours Summer-work
ALGEBRA II Notes & Formulae [Please study and master!]

1. If a function f(***x***) is given, then, for f(***a***), substitute ***a*** for ***x*** and simplify!

2. **Slope** of a line, m = Rise / Run = $(Y_2 - Y_1)/(X_2 - X_1)$ = Subtract the Ys / Subtract the Xs

3. The general **vertical line** equation is: x = **a** [any #], and the general **horizontal line** equation is: y = **b** [any #].

4. A *rising* line has a **positive** slope; a *falling* line has a **negative** slope; a *horizontal* line has a slope of **zero**; a *vertical* line has an **undefined** slope.

5. Two lines are *parallel* if their slopes are the **same**; two lines are *perpendicular* if their slopes are **negative reciprocals**.

6. Equation of a line in **Slope Intercept Form**: Y = mX + b. To *graph*, locate the y-intercept, *b*, first, and then adjust for the slope, *m* = rise / run

7. Equation of a line in **Point Slope Form**: $Y - Y_1 = m(X - X_1)$

8. To graph a line in **Standard Form**: aX + bY = C, find and plot the *x*- and *y*-intercepts first, then draw the line!

9. Given 2 Points, to find equation of a line, find slope, m, first using #2; then, use #7.

10. Factoring / Expansion:
   a. $(a - b)^2 = a^2 - 2ab + b^2$
   b. $(a + b)^2 = a^2 + 2ab + b^2$
   c. $(a - b)(a + b) = a^2 - b^2$
   d. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
   e. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

11. **Complex Numbers**: a + b*i*
a) Addition / Subtraction: Simply combine Like Terms!
b) Multiplication: Apply Distributive Law [/ FOIL] keeping mind $i^2 = -1$
c) Division: For (a + b*i*)/(c + d*i*), multiply top & bottom by the *conjugate* of the denominator [i.e. c – d*i*]. Then simplify the numerator and denominator.

12. **To get rid of fractions** in an equation, multiply throughout by the LCM of the denominators.

13. **Square Roots**:
   a. $\sqrt{a}\cdot\sqrt{b} = \sqrt{(a\cdot b)}$
   b. $\sqrt{(a/b)} = (\sqrt{a}) / (\sqrt{b})$
   c. If $x^2 = a$, then $x = \pm\sqrt{a}$; if *a* is negative, then $x = \pm i\sqrt{a}$.

14. For the quadratic equation, $ax^2 + bx + c = 0$, the **Discriminant, D** = $b^2 - 4ac$.
   a. If D = 0, the quadratic equation has 1 *real* root.
   b. If D > 0, the quadratic equation has 2 *real* roots.
   c. If D < 0, the quadratic equation has 2 *complex* roots.

15. The **Quadratic Formula** to solve the quadratic equation, $ax^2 + bx + c = 0$ is:
$x = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$

16a) The **maximum / minimum value** of a quadratic function in **standard form**, $y = f(x) = ax^2 + bx + c$ is given by its **Vertex**, for which $x = -b/2a$. To find *y*, substitute $x = -b/2a$ into the function, y = f(x).
b) The Vertex of the quadratic function in Vertex Form, $y = a(x - h)^2 + k$ is given by V(h, k). That is, for the *x*, flip the inside # in the ( ); for *y*, just take the outside #!

17. For the ***x*-intercepts** of a function, y = f(x), set y = 0 and solve. For the ***y*-intercepts** of a function, plug in x = 0.

18. **N-th root:** If $x^n = a$,
   a. *if n is odd,* then $x = a^{1/n}$ b. *if n is even,* then $x = \pm(a^{1/n})$.
19. To find the **Inverse** of Function, y = f(x)
   a. Switch *x* and *y* i.e. x = f(y)

b. Solve for y!

20. **Logarithms**:
   a. If $a^b = c$, then $\log_a c = b$
   b. If $\log_p q = r$, then $p^r = q$ *or* $q = p^r$
   c. *ln* ~ $\log_e$
   d. Calculators operate with log (base 10) and *ln* (base *e*) modes.

21. **Log Rules**: *Irrespective of the <u>base</u>,*
   a. $\log_a a = 1$ so that *ln* e = 1 [see #20 c. above]
   b. log 1 = 0
   c. log x·y = log x + log y
   d. log (x/y) = log x – log y
   e. $\log (x)^y = y \cdot (\log x)$

22. **Change-of-Base Formula** for Logarithms: $\log_b a = \log_{10} a / \log_{10} b$

23. Solving **Exponential Equations**:
   a. Isolate the exponent term
   b. Express in *log* form [#20 a. above]
   c. Use Change-of-Base Formula [#22 above] or *ln*, if applicable, to solve.

24. Solving **Logarithmic Equations**:
   a. Isolate the *log* term
   b. Express in exponential form [#20 b. above]
   c. Solve.

25. **Log Rules**
a. log (*a*·*b*·*c*) = log *a* + log *b* + log *c*
b. log (*a*/*b*) = log *a* – log *b*
c. $\log (a)^b = b \cdot \log a$

26. **Continuous Compounding**: $A_t = Pe^{rt}$ or $A_t = A_o e^{rt}$ where P [or A_o] denotes the Initial Amount, r ~ rate of growth [in decimals!] and t ~ time

27. **Compound Interest Formula:** $A_t = P(1 + r/n)^{nt}$ or e^{rt} or $A_t = A_o (1 + r/n)^{nt}$ where P [or A_o] denotes the Initial Amount, r ~ rate of growth [in decimals!], n ~ number of times the amount is compounded in 1 year, and t ~ time

28. **Distance Formula**: $d = \sqrt{[(X_2 - X_1)^2 + (Y_2 - Y_1)^2]} \sim \sqrt{[(\text{Subtract Xs})^2 + (\text{Subtract Ys})^2]}$

29. **Mid-Point Formula**: $M[(X_1 + X_2)/2, (Y_1 + Y_2)/2] \sim M(\text{Average of the Xs, Average of the Ys})$

<u>Points to Remember for SUMMER-WORK</u>
1. This summer-work for Trig. Precalculus Honours is *not* a "punishment"! It is an "honour" to be in the class: you are amongst the brightest and most diligent of Algebra II students. As the title implies, the Trig. Precalculus Honours class is a ready *gateway* for the AP Calculus BC with Ms. Stip [or to a college Calculus class]. Some of you exceptionally bright and hard-working chaps may choose to do AP Calculus along with AP Statistics that *I* teach!

2. The Summer-work has been structured so as to give you practice for vital and basic skills that I *assume* that you'd possess in the Trig. Precalculus Honours class. I would understand if you've forgotten a few Algebra II concepts; but the objective of the summer-work is to refresh in your minds the *most* critical of these Algebra II ideas.

3. The Precalculus Honours summer-work is *not* optional: it is *part* of your incoming grade for the Honours class.

4. There shall be a Test held in the 1st week of school *based* on this summer-work.

5. <u>Most</u> Qs *require* work to be shown for **full** credit: failure to do so shall produce a ZERO for those Qs.

6. I shall endeavour to send you the answers to the Summerwork Qs so you can check your answers! Again, I grade based on the *process* [i.e. procedure], not the *product* [i.e. the end result / answer] alone.

7. Check your email frequently over the Summer for Notifications / Tips / Suggestions and Jokes!

8. Feel free to email me in case of doubts, Qs and clarifications. **Before that, do consult a friend or the Internet for the concept and examples clarifying the concept, yes?**

9. I strongly encourage you to do the summer-work with friends also in the Trig. Precalculus Honours class: it shall strengthen relationships and enable you to consult each other!

Duration

a. Most **Page** assignments below are *designed* to take ~15 min, many just 5-10min.! So do not feel "threatened" by the "enormity" of it.
b. The Summer-work *shouldn't* take more than 7-10 days to do, allocating ~ an hour / day. It constitutes a fairly thorough review of Algebra II.
c. **Recommendation:** PLEASE work on the Assignment *as soon as possible* while the material is *fresh* in your minds. Doing the work towards the end shall result in a shoddy job due to you having forgotten many of the concepts!

Format of Summer-work

1. ***As far as possible***, each assignment is to be done on a *separate* page [*not* separate sheet: you are encouraged to write on BOTH sides of a sheet! I just want a new assignment topic on a new page!]. However, if the Solutions / Answers to a certain page only consumes the TOP-HALF of a side, then you may continue do the *next* assignment on the BOTTOM HALF [I hate wasting paper!]

2. Label EACH Assignment clearly on the Margin.

Assignment	Page	Question #	Topic
1	16	#57-60 **all** **[Tip!** Define a variable *x* to write the required expressions.]	Interpreting Word Problems
2	22	#11, 12, 33-36 **all** [**Tip!** Distribute *very* carefully first (if necessary), then get rid of fractions by multiplying throughout by the LCM of the denominators] **Extra Credit:** #45, 47 **[Tip!** Define a variable *x* and write, then solve an equation.]	Solving Equations; Interpreting Word Problems
3	24	#69-77 **odd** PLUS 72, 74, 78 [Show *each* step.]	Simplifying Expressions
4	29	#5-9 **odd** [**Tip!** Get rid of fractions by multiplying throughout by the LCM of the denominators] #24-28 **even**	Solving for a specified variable
5	45	#27-47 **odd** [**Caution!** When multiplying / dividing by a *negative* number, flip the inequality in the *next* step.]	Solving Inequalities
6	72	#43-50 **all**	Evaluating Functions
7	79	#7-13 **odd**	Slopes of Lines
8	86	#5-35 **odd**, #43-57 **odd**, #75-79 **all**	Graphing Lines in Slope Intercept and Standard Form
9	95	#5-11 **all** [**Tip!** Wherever possible, leave your answer in Point-Slope Form: $y - y_1 = m(x - x_1)$] #13-17 **odd** #19-25 **odd** #26 #27 [**Tip!** Sketch a figure illustrating the situation first.]	Writing Equations of Lines
10	111	#5-12 **all** [**Tip!** You *may* use a test-value – usually but not always: (0, 0) - to decide which region to shade!]	Graphing Inequalities
11	117	#5-8 **all**, # 13-20 **all**	Evaluating Piecewise-Functions

		Extra Credit: #21, 22, 24, 26 [**Tip!** * Graph the 1st line * Identify and plot the "critical point" on the line - e.g. in #23: (given) x = -5, (by substitution) y = 3 * Shade the line to the left or right of the critical point depending on the inequality. Erase "unwanted" portion. * Repeat procedure for the 2nd line	
12	153	#35, 37, 39, 45	Solving Systems of Equations
13	260	#4-9 **odd**, #23-35 **odd**, #38-40 **all**, #47-52 **all**, #56-62 **all [Tip!** These are 2-step factoring Qs!] #81-85 **odd**: find the x-intercepts of the parabola *after* factoring [**Tip!** Set y = 0, and solve. Factor out the *negative* sign, first, if necessary.]	Factoring & Solving Quadratic Equations By • Taking a Monomial out • Trinomial Method • Difference of Squares: $a^2 - b^2$
14	267	#5-11 **odd**, #13-67 **odd** [**Tip!** Isolate the square term ( )^2 first, then take square-roots: $X^2 = k \Rightarrow X = \pm k$]	Solving Quadratic Equations by Taking Square Roots
15	277	#17-27 **odd**, #37-61 **every other odd** [as in 37, 41, etc!]	Complex Numbers
16	295	#17, 19, 23 #56-59 **all [Tip!** See Box on P 293]	Using the Discriminant and the Quadratic Formula
17	336	#91-95 **odd** #96-100 **all** [**Tip!** Write as $y = ax^2 + bx + c$ i.e. without any ( ). **Caution!** Watch the *negative* sign outside, if present!] #103-109 **odd [Tip!** Use any method: * Take square-roots immediately **or** ***** Get all terms to one side, factor and solve **or** * Get all terms to one side, and use Quadratic Formula.]	Simplifying expressions Solving Quadratic Equations
18	341	#21, 25, 27, 31, 35, , 45, 53, 55, 56, 58, 61	Simplifying Polynomials
19	356	#19-31 **every OTHER odd** [as in 19, 23, etc.!] [**Tip!** If a certain term, say, x^2, is missing, you *must* write it as $0x^2$.]	Dividing Polynomials by Long / Synthetic Division
20	376	**ONLY** find the x-intercepts and y-intercepts for #4-7, 13-22 **all** [**Tip!** The quadratic expression in #19-22 has complex roots…so may be disregarded for x-intercepts! **Caution!** Show work.]	Finding *x*- and *y*-intercepts of polynomials.
21	405	#53-61 **odd** [**Tip!** Show preliminary work, and use a calculator! Isolate the Power term first, then take *n*-th roots. For even powers, recall $X^n = k \Rightarrow X = \pm(k)^{1/n}$.]	Solving Power Equations [n-th root]
22	426	#15-23 **odd [Tip!** Switch x and y. Rewrite the expression so that the y is on the left. Then solve for y. If necessary, multiply throughout by the LCM of the denominator, *first*, to get the rid of the fraction!] #36, 39, 40, 42, 43, 45	Finding Inverse of Functions
23	490	**[**Consult the above **Algebra II Notes** #20-25 for a refresher on *logs*.] #17-23 **odd** #24-35 **all** [**Tip!** Set each expression = *x*, then rewrite the equation in exponential form. Finally, use common-sense / make the bases the same and solve for *x*.] #37-47 odd [**Tip!** Use a calculator!]	Using Definition of Logarithms
24	496	#31-51 **every other odd [**as in #31, 35, etc!], 59-65 **odd [Tip**! Use a calculator.] **Extra Credit:** #14-21 **all**	Using Log Rules to Expand / Condense expression

		[Tip! Method I Set each expression = *x*, then rewrite the equation in exponential form. Finally, use common-sense / make the bases the same and solve for *x*. Method II Use Method I along with log rules]	
25	505	**[**Consult the above **Algebra II Notes** #20-24 for a refresher on Solving *logs* Equations.] #31-35 **odd**, 41 #44-49 **all**	Solving Log and Exponential Equations
26	558	#3-11, 17-41 **every OTHER odd** [as in 3, 7, etc.!]	Multiplying & Dividing Rational Expressions
27	566	#27-35 **odd**	
28	592	#5-13 **every other odd** [as in 5, 9, etc.!]	